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Color scaling of discs and natural objects at different luminance levels

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Abstract

Assigning a basic color name to an object and rating the amount of a particular hue is a fundamental visual capability. Traditional color scaling studies have used increment flashes or isoluminant stimuli of a homogeneous color. Natural objects, however, do not contain a single color but are characterized by a distribution of different chromatic hues. Here we study color scaling using photographs of natural fruit objects. Stimuli were either homogeneous spots, digital photographs of fruit objects (e.g., banana), or outline shapes of the fruit objects. Stimuli were displayed on a CRT monitor on a homogeneous white background; its luminance was varied above and below the medium gray. The chromaticity of the stimuli was varied in 36 equally spaced chromatic directions in the isoluminant plane of the Derrington-Krauskopf-Lennie (DKL) color space. For each stimuli, subjects rated the amount of red, green, blue, and yellow in the stimulus on a scale from 0–8. In agreement with earlier studies we found that the positions of the peak ratings for each color do not coincide with the cardinal axis of DKL color space and are largely invariant under changes of the background luminance. For the average rating we found a dependence on background luminance for all colors: yellow ratings increase with darker backgrounds, whereas ratings for the other colors, in particular green, decrease. For the fruit objects, we found a selective increase in the average color rating for the natural fruit color. For example, the average rating for yellow was 1.7 times higher for the banana images compared to disc stimuli. No such selective increase was found for outline shapes. We conclude that the distribution of hues in natural objects with a characteristic object color can have a profound effect on color scaling and color appearance.

Keywords: Color vision, Color scaling, Natural images, Unique hues, Color appearance

Introduction

Human color vision begins with the transduction of electromagnetic radiation with wavelengths ranging from about 380–700 nm by three different types of photoreceptors in the retina. The three different types of cone photoreceptors differ by their peak absorption rate at short (S), middle (M) and long (L) wavelengths. The absorption spectrum of L and M cones cover almost the entire band of visible wavelengths and are similar, differing only at higher wavelengths. Already at the level of the retinal ganglion cells, the signals from the three cone types are combined in three cone-opponent channels to form a more efficient, decorrelated coding of the visual input. One channel is a luminance channel, where L and M cone input is additively combined (L + M). The other two channels are chromatic channels, one coding the difference between L and M cones (L – M), the other the difference between S cones and summed L and M cones [S–(L + M)]. These three cone-opponent channels have been identified physiologically and psychophysically, and define the axes of the Derrington-Krauskopf-Lennie (DKL) color space (Derrington et al., 1984; Krauskopf et al., 1982). At the cortex, these signals undergo further transformation [for a review, see Gegenfurtner (2003)], until ultimately a color name such as red, green, blue or yellow can be assigned to a particular color. Early studies tried to directly relate, in a truly psychophysical spirit, the color sensation to the stimulus parameters and a linear combination of cone inputs (Schrödinger, 1925; Hurvich & Jameson, 1955). For a review, see Valberg (2001). In the seminal study by Hurvich and Jameson (1955), a hue cancellation technique has been used to determine the wavelength of the unique colors. In these early studies stimuli were confined to monochromatic spectral lights. More recently, isoluminant stimuli have been used, which allow to study extra spectral purplish hues and to relate color scaling to the properties of cone-opponent cells in the LGN (De Valois et al., 1997). As suggested by De Valois et al. (2000), we use the term "color scaling" instead of "hue scaling" to acknowledge the observation that the scaling is sometimes as much akin to hue scaling as to saturation scaling. A number of studies have than investigated the relation between the properties of the early chromatic stages of the visual system and color naming using a color-scaling paradigm (De Valois et al., 2000; Webster

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et al., 2000; Wuerger et al., 2005). These studies have shown consistently that the cone-opponent channels of the DKL color space differ considerably from the perceptually opponent channels black–white, red–green, and blue-yellow as proposed by Hering (1964). Though this difference has been stated even in the first physiological study of the cone-opponent channels (De Valois et al., 1966) and since then has been confirmed in various color scaling experiments, the cone-opponent channels are still commonly referred to as red-green and blue-yellow color opponent channels.

Previous color scaling studies have used isoluminant stimuli of a homogeneous color (De Valois et al., 1997, 2000; Malkoc et al., 2005; Webster et al., 2000; Wuerger et al., 2005). In this study, we extend this work in two directions: first, we used homogeneous patches at different luminance values to investigate the stability of color categories across luminance variations. Previous studies have found a consistent deviation between the cone-opponent channels and the loci of the unique hues using isoluminant stimuli of an intermediate luminance level. In the isoluminant plane, blue is too light and yellow too dark compared to colors, which are usually called "blue" or "yellow," and this may affect the deviation of the $S-(L+M)$-axis from blue-yellow. Does the use of stimuli with a considerable luminance difference from the background change the loci of the unique hues? Our results show that this is not the case and that the position of the peak rating of each color category differs only slightly, depending on the luminance of the stimuli relative to the background. Second, we used stimuli of natural objects with a characteristic object color, such as a yellow banana, to investigate the influence of the object color on the color rating. The experiments reveal a profound effect of the object color on color rating, such that a banana receives higher ratings of yellow compared to homogeneous patches of the same chromatic direction.

Materials and methods

Apparatus

Software for the presentation of the stimuli was programmed in C using the SDL library. The stimuli were displayed on a SONY GDM-20se II color CRT monitor, which was viewed binocularly at a distance of 0.40 m in a dimly lit room. The monitor resolution was set to 1280 × 1024 pixels with a refresh rate of 120 Hz non-interlaced. The monitor was controlled by a PC with a color graphics board with 8-bit intensity resolution for each of the three monitor primaries. For each primary, the nonlinear relationship between voltage output and luminance was linearized by color look-up tables. To generate the three look-up tables, the luminances of each phosphor were measured at various voltage levels using a Graseby Optronics Model 307 radiometer with a model 265 photometric filter, and a smooth function was used to interpolate between the measured data. A Photo Research PR 650 spectroradiometer was employed to measure the spectra of each primary at maximum intensity. The spectra were multiplied with the Judd-revised CIE 1931 color matching functions (Judd, 1951; Wyszecki & Stiles, 1982) to derive CIE xyY coordinates of the monitor phosphors (Irtel, 1992). In the following, luminance and photometric luminance refer to the $V(\lambda)$ curve as modified by Judd (1951). The xyY coordinates of the monitor primaries at maximum intensity are given by $R = (0.613, 0.349, 20.289)$, $G = (0.283, 0.605, 64.055)$, and $B = (0.157, 0.071, 8.631)$. Cone contrasts were computed from the spectral distribution of the monitor primaries using the cone fundamentals of Smith and Pokorny (1975).

Color space

The stimuli were defined within the isoluminant plane of the DKL color space (Derrington et al., 1984; MacLeod & Boynton, 1979). The DKL color space is a spherical color space spanned by the three second stage cone-opponent axes, the two chromatic axes $L - M$ and $S-(L + M)$ and the achromatic axis $L + M$ (Fig. 1a). The three axes define the "cardinal" direction of the DKL color space and intersect at the white point. The two chromatic axes define the isoluminant plane. The DKL color space is a linear transformation of the LMS cone contrast space (Brainard, 1996). Along the $L - M$ axis, the excitation of the S cones is constant whereas the excitation of L and M cones covaries such that their sum is constant. Color along the $L - M$ axis changes from blue-greenish to reddish. Conversely, along the $S-(L + M)$ axis, only the excitation of the S cones changes, whereas the excitation of the L and M cones remains constant. Color along the $S-(L+M)$ axis changes from yellow-green to purplish. Within the isoluminant plane, colors are defined by their chromatic direction given by the azimuth ranging from 0° to 360° and their chromatic contrast given by the distance from the white point.

The length of a vector that lies along a particular half axis can be described in terms of the contrast delivered to the cone mechanisms (Smith & Pokorny, 1975). By modulating lights along the $L - M$ axis, the maximal L and M cone contrasts were 9% and 17%, respectively. By modulating lights along the $S-(L+M)$ axis, 85% contrast to the S-cones could be achieved. Our values are in close agreement with other studies (e.g., De Valois et al. (2000) reported 9%, 17%, and 83% L-, M- and S-cone contrast). The maximum values along each axes are set to unity, and the intermediate angles were defined as combinations of these unit vectors. Chromatic excursions in different intermediate directions produce sinusoidal variations in cone activation for each cone type (Fig. 2). These sinusoidal variations result from the linear projection of the chromatic stimuli to the cone mechanisms. Because there is no official standardization of DKL color space, the plot of the cone contrast curves allows a quantitative comparison of the chromatic directions with other studies.

Stimuli

Three different classes of stimuli were used: discs, digital images of fruits, and outline shapes of the fruit stimuli. Three different fruit images were used (banana, lemon, and lettuce), which were selected because of their distinctive object color (yellow or green). The stimuli were shown at different chromaticities and were defined in the DKL color space. The chromaticities of the stimuli were isoluminant shifts from the white point in 36 chromatic directions, from 0–360° in steps of 10°. For the natural images, the whole distribution of different chromaticities was shifted in the isoluminant plane along the $S-(L + M)$ and $L + M$-axes in the particular direction (Fig. 1). More precisely, the RGB image was transformed to DKL space, a constant offset Δ_{L-M} and $\Delta_{S-(L-M)}$ was added to the activity in the $L + M$ and $S-(L+M)$ planes, respectively, and the image was transformed back to RGB space. The mean chromatic contrast of the stimuli was set to 1.0 (i.e., the maximum available for the monitor primaries). Stimuli were presented on a homogeneous gray background at five different luminance levels. The luminance in DKL space is specified as elevation above (positive angles) or below (negative angles) the isoluminant plane (0°). We used 5 luminance values at elevations −90°, −30°, 0°, 30°, and 90° with corresponding luminances of 0, 23, 46, 69,

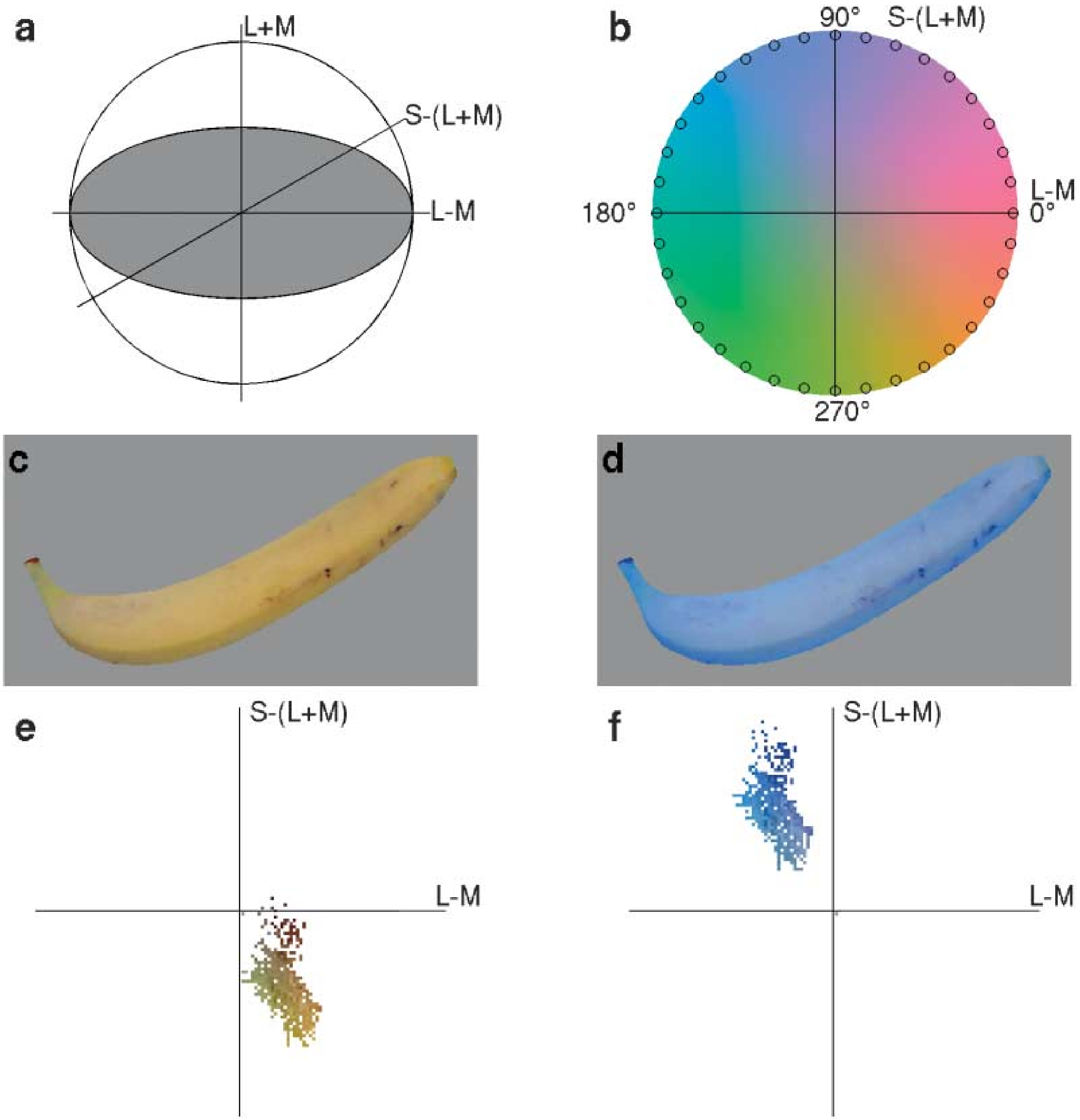

Fig. 1. a: DKL space with the isoluminant plane shown in gray. The isoluminant plane is spanned by the L − M and S–(L + M) axes, which together with the achromatic L + M axis define the cardinal axes of the DKL color space. b: display of the isoluminant plane. The projections of the mean chromaticities of the stimuli used in the present experiments to the isoluminant plane are indicated by open circles. c and e: banana image and its projection to the isoluminant plane. d and f: a colored banana is synthesized by shifting the original distribution in the isoluminant plane.

and 92 cd/m^2. The fruit stimuli were adjusted to have a mean luminance that coincides with the white point (46 cd/m^2). The standard deviation of the luminance in the stimuli was 8 cd/m^2 for the banana and lemon and 14 cd/m^2 for the lettuce.

Paradigm

We used the same color scaling procedure as employed in a former study (De Valois et al., 2000). For each of the four colors red, green, yellow, and blue observers signaled the amount of that color in the present stimulus on a scale from 0–8 by pushing an assigned button. After each rating, pressing another button initiated the presentation of the next stimulus. On a given run, each stimulus was presented twice in random order. For the same stimulus, five runs at different luminance values of the background were usually completed by the observers in a single session. Sessions for different kinds of stimuli were interleaved, and each session was repeated 5 times for a total of 10 judgments for each chromatic

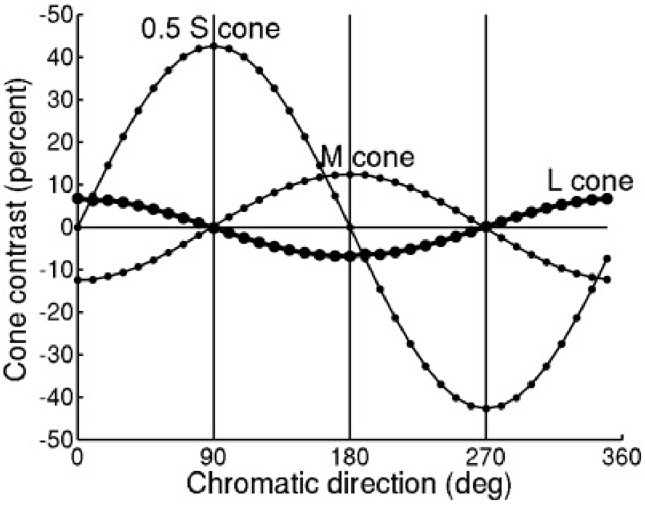

Fig. 2. Percentage of changes in L, M, and S cone activation produced by the 36 disc stimuli.

direction. Sessions were often distributed over periods of several weeks to prevent observers from recalling their previous judgments when the particular chromaticity was presented again.

Observers

Five observers participated in the study, two male (FS, MM) and three female (AF, SB, TF). All observers were naïve as to the purpose of the experiment. All had normal color vision and normal or corrected-to-normal visual acuity. No systematic differences between the observers were found. Due to the limited availability of observers, not all experiments were run by all observers. In particular, the experiments with the outline shape of the lemon were run only by SB. All experiments were conducted in accordance with principles embodied in the Declaration of Helsinki (Code of Ethics of the World Medical Association).

Results

The Results section is organized in two main sections: in the first section, the effects of luminance variations on the color scaling functions are analyzed. In the second section, we compare scaling functions for standard disc stimuli with scaling functions for natural images.

Variation of background luminance

How does the relative luminance of an object influence the rating of the four unique colors red, green, yellow, and blue? Fig. 3 shows the color scaling results obtained for a 2° foveal disc stimulus at different luminance levels of the background. From the total of five luminance levels investigated we show three, namely the medium level at 0° (isoluminant plane) and the extremes with an elevation of −90° (black) and 90° (white). Results were similar for each of the three observers, so we present averaged data. Overall, the curves are largely similar for the different luminance values. Only yellow shows a pronounced increase with relative higher luminance of the stimulus compared to the background.

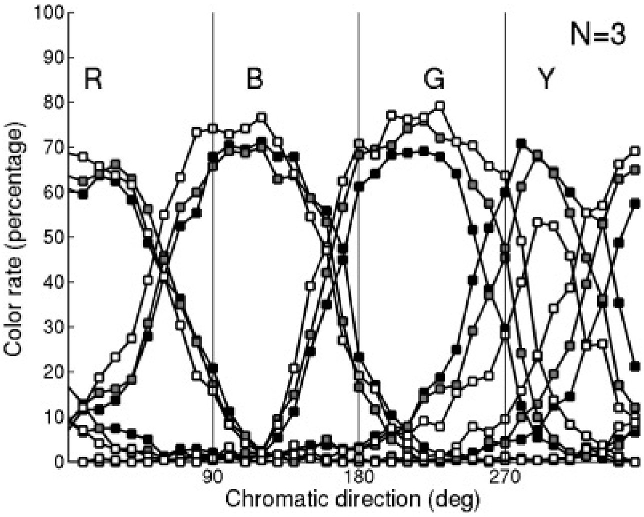

Fig. 3. Color scaling results for a 2° foveal disc stimulus at 3 different relative luminance levels with elevation −90° (black square), 0° (medium gray square), and 90° (light gray square). The two intermediate elevations of −30° and 30° are omitted for clarity of the plot. Results are averaged across three observers, each of which made 10 ratings at every chromatic direction.

Next we analyzed how the locations of the maximum rating depend on the luminance of the background (Fig. 4, left). In agreement with previous reports (De Valois et al., 1997, 2000; Webster et al., 2000; Wuerger et al., 2005), we found that the location of the maximum rating does not coincide with the cardinal axes of DKL color space. Further, the location of the maximum rating of the four colors remains constant under large variations of the relative luminance of the stimulus. For the robust computation of the maximum, the rating functions are smoothed with a Gaussian of SD 20°. The chromatic directions of maximum ratings for the four basic colors are 20° (red), 224° (green), 304° (yellow), and 124° (blue). With relation to the cardinal axes of DKL color space, the unique hues thus have offsets of +20° (red), +44° (green), +34° (yellow), and +34° (blue).

Next, in order to more quantitatively investigate the dependence of the overall rating on the relative luminance level, we compute the mean rating for each color at the different luminance levels. The mean rating is the average rating across all 36 chromatic directions. Results are shown in Fig. 4, right. With increasing luminance of the background, yellow shows the largest effect, with a decrease of the mean rating from 21% to 12% (−9%), whereas the other colors show an increase of the mean rating. This increase is larger for green (+7%) than for red (+5%), than for blue (+2%). In other words, green and red ratings are higher for stimuli darker than the background, and yellow ratings are higher for stimuli brighter than the background. Ratings for blue are largely independent of the relative luminance.

Natural images

All previous studies on color scaling have used stimuli of a single homogeneous color. Natural objects, however, are characterized by a distribution of different chromatic hues, and have often a single characteristic color category that is associated with this object

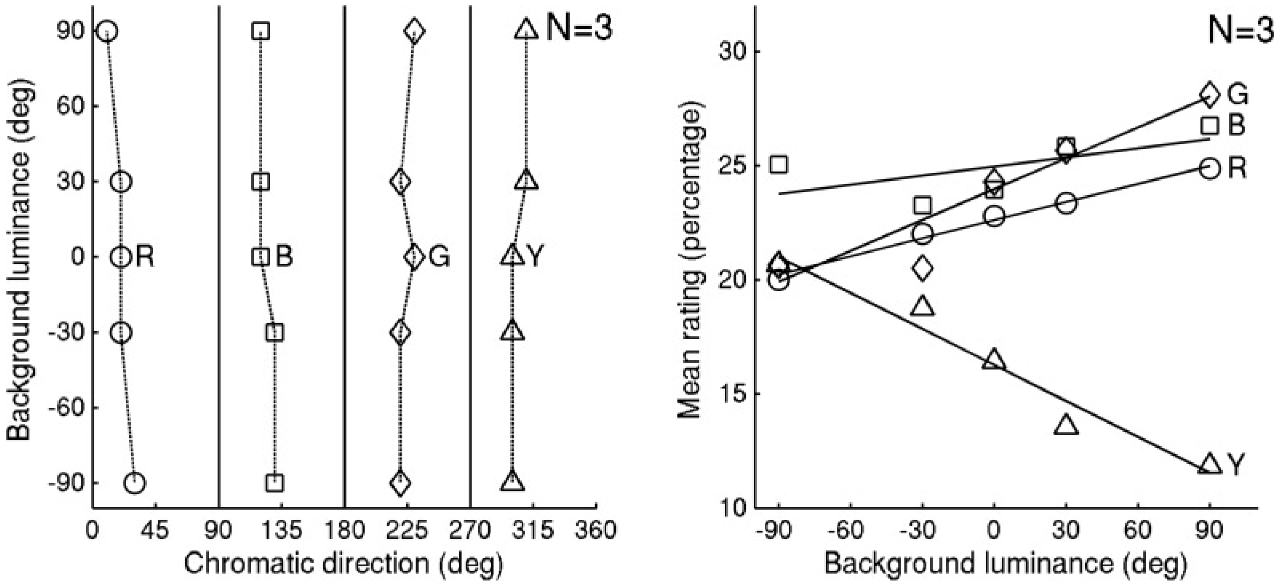

Fig. 4. Left: location of the maxima in the color scaling functions at the five different luminance levels of the background. For each color, maxima show little variation across different background luminances. Right: variation of the mean color ratings for the four different luminance levels of the background. For yellow, average rating decreases with increasing background luminance, whereas for green, red, and blue average rating increases with increasing background luminance.

(e.g., a yellow banana). Here we investigate color scaling using digital images of fruits.

The color scaling functions for a banana image for three observers and the averaged data across these observers are shown in Fig. 5. Compared to the color scaling functions for a standard disc stimulus, all observers show a pronounced increase of the yellow rating, accompanied by a decrease of the adjacent colors green and red, and no effect for the blue rating curve.

In order to quantify the relative increase in the overall rating, we computed for each color an index based on the ratio between the mean rating $\mu(\mathrm{r})$ for the fruit and for the disc

$$i_{\text{color}} = \mu(r_{\text{fruit, color}})/\mu(r_{\text{disc, color}}) - 1. \qquad (1)$$

This measure is positive if the average rating is higher for the fruit compared to the disc, negative if the rating is lower and zero if both types of stimuli receive the same average rating for the particular color. Results of applying this measure are shown in Fig. 6. For the banana, the strongest effect occurs for yellow with an index of 0.8. This value is close to an index of 1.0, which corresponds to a doubling in the mean rating. In the data averaged across observers, both red and green have a smaller negative index reflecting a small decrease in the average rating for the banana. The index for blue is close to zero (i.e., no differences occur between the blue ratings for the banana as compared to the disc). A similar selective increase in the rating of the object color is observed for other fruits (Fig. 7).

The natural stimuli differ in at least two important aspects from the disc. First, they have a particular shape, which allows identification of the object and associate a particular color. Second, they have color textures of different chromatic hues. In order to investigate the effect of the object shape on the observed shift, we replicated the color scaling with a homogeneously colored outline shape of the banana (Fig. 6). For observer SB, the outline shape of the banana leads to an unspecific increase for every color rating, in contrast to the highly specific pattern of increases and decreases obtained for the banana image. For observers AF and MM, the outline shapes of the banana lead only to a small increase of the yellow rating. Because the observers reported the outline shape as clearly detectable as a banana, we conclude that higher level, cognitive influences alone cannot explain the increased ratings.

Finally, we have also determined for the banana stimulus the dependence of the average rating and the variation of the location of the unique hue (Fig. 8). The locations of the unique hues are similar for banana and disc stimuli (Fig. 8, left). For the banana stimulus, the average chromatic directions of maximum ratings for the four basic colors are 26° (red), 210° (green), 300° (yellow) and 118° (blue). For the average rating (Fig. 8, right), the slopes of the curves are similar to the results obtained for the disc. Yellow decreases with increasing luminance, green increases, and blue are largely constant. The only difference occurs for red, which is constant for the banana but increases for the disc. Overall, the natural banana images show a smaller dependence on background luminance than the discs.

Discussion

In this study we have investigated color scaling for the unique colors red, green, yellow, and blue at various luminance levels. Whereas previous studies exclusively used synthetic stimuli of a single homogeneous color, we also used natural images of fruits as stimuli.

Previous studies agree that the unique hues do not coincide with the cardinal axes of DKL color space (De Valois et al., 1997, 2000; Webster et al., 2000; Wuerger et al., 2005). In these studies, the stimuli are often isoluminant to the middle gray background, resulting in stimuli of medium luminance, which may differ considerably from, e.g., a prototypical bright yellow. By presenting stimuli at different relative luminance levels we investigated whether this may cause a rotation of the unique hue maxima towards the

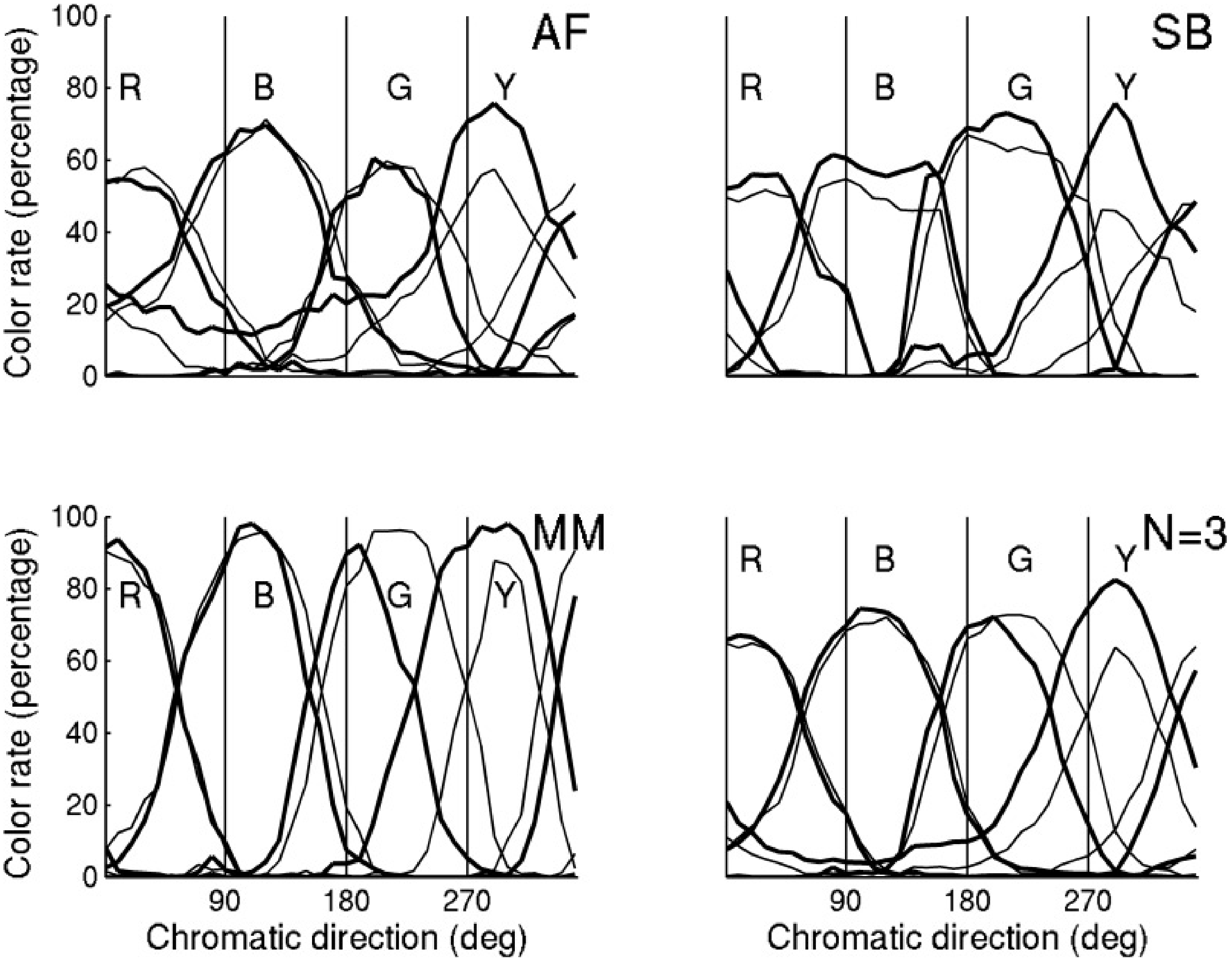

Fig. 5. Color scaling of banana stimuli (bold line) compared to disc stimuli (solid line) for individual observers (AF, MM, SB) and averaged across observers (N = 3). Results for the individual observers (AF, MM, SB) are averaged across 10 ratings at 5 luminance values of the background. For all observers, there is a pronounced increase of the yellow rating accompanied by a decrease for the adjacent categories of green and red. No differences occur between the blue ratings.

cardinal axes. However, this was not the case. In accordance with previous studies we found that the locations of the unique hues are largely invariant under luminance changes (Larimer, 1974; Larimer et al., 1975), and they do not coincide with the cardinal axes (De Valois et al., 1997, 2000; Webster et al., 2000; Wuerger et al., 2005). In addition to the location of the maximum we have also analyzed the luminance dependence of the average rating. For the average rating we found that yellow receives higher ratings at

Fig. 6. Rating index based on the ratio of the mean rating for the banana stimuli (solid bars) or outlines shapes (open bars) compared to disc stimuli. In the averaged data for the banana stimuli (N = 3) there is a pronounced increase for the rating of the characteristic object color yellow (Y), accompanied by a decrease of the adjacent color categories red (R) and green (G). No effect is found for the opponent color blue (B). The average rating index for outline shapes of the banana (open bars) shows only a small increase for yellow, with little change of the other categories.

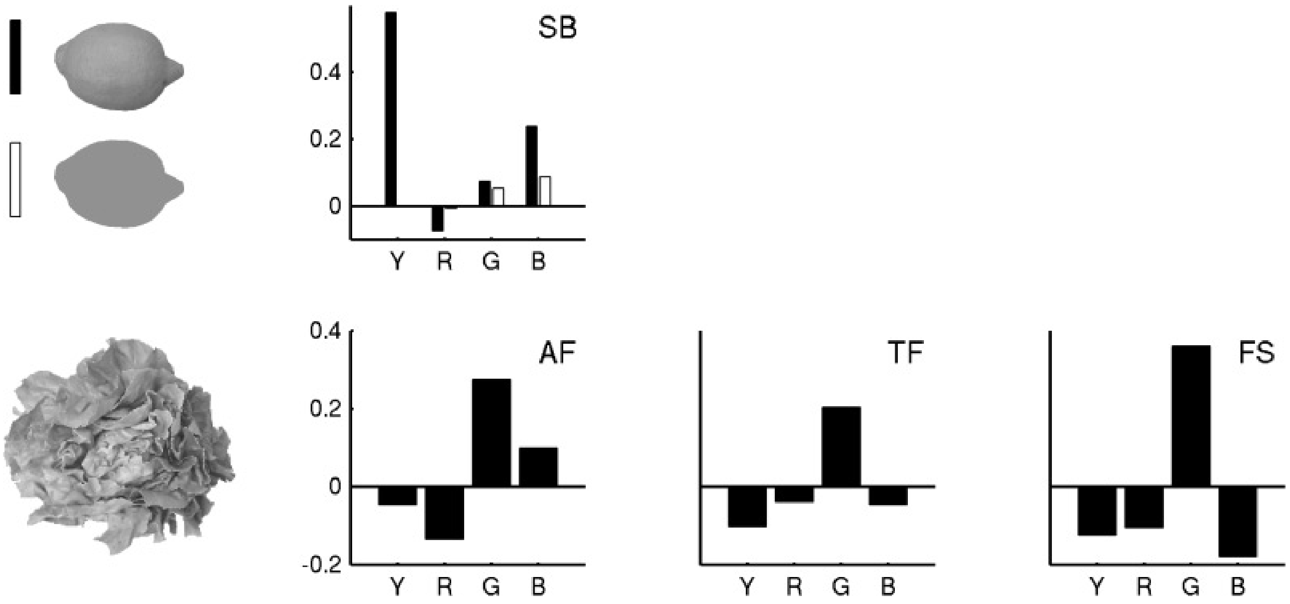

Fig. 7. Top: Rating index for the lemon stimulus (top, solid bars) shows a selective increase of yellow (Y). Bottom: For the lettuce stimulus (solid bars) green (G) consistently receives a higher rating than the other colors yellow (Y), red (R), and blue (B).

higher relative luminance, whereas the opposite is true for green, red, and blue (in decreasing magnitude).

Further, we have investigated color scaling of natural images with a characteristic object color such as a yellow banana. Here we found for the characteristic object color an overall increase of the average rating. At the same time, the average color rating for adjacent color categories is often reduced, whereas the opponent color remains largely unaffected.

Different causes may have led to the observed shifts, which can be roughly divided into sensory and cognitive effects. For example, a sensory bias could occur because the distribution of the different hues in the banana varies mostly along the blue-yellow axes. Shifting this distribution such that it has the same average yellow as the disc stimulus effectively results in at least some yellowish colors of higher saturation, which may cause the increase in the yellow response. However, because the same increase

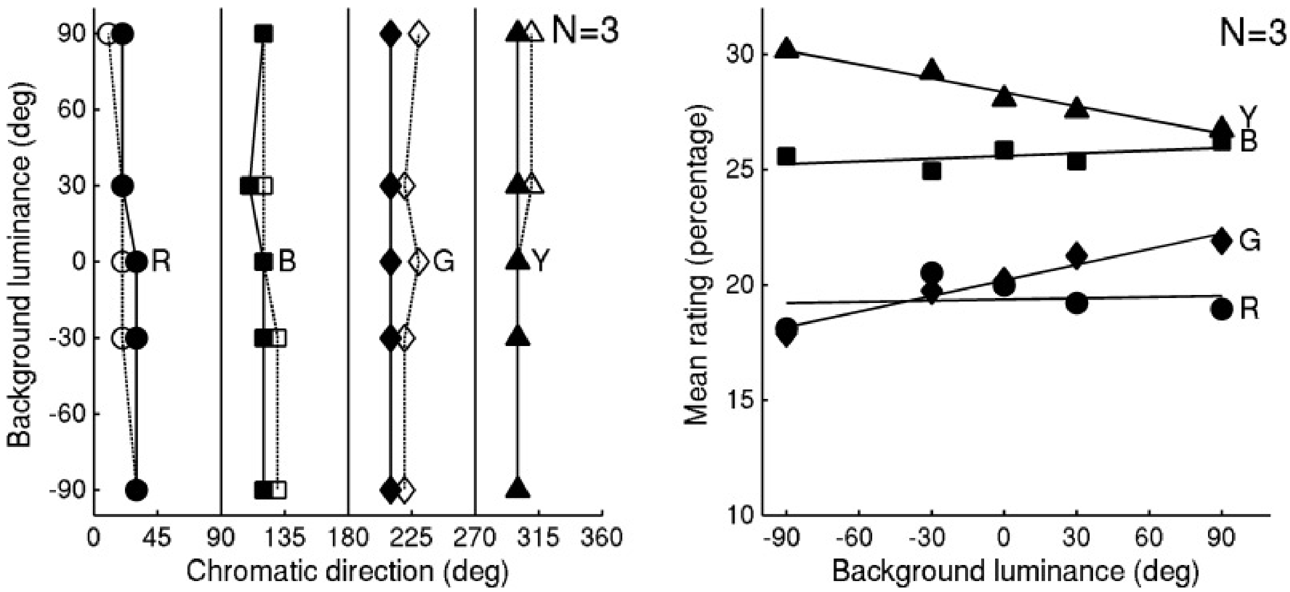

Fig. 8. Left: location of the maxima in the color scaling functions at the five different luminance levels of the background. Similar to the disc stimuli (dashed lines), maxima for the banana stimuli (solid lines) show little variation across different background luminances. Data averaged across three observers. Right: variation of the mean color ratings for the five different luminance levels of the background for the banana stimulus. Ratings for yellow decrease with increasing background luminance, whereas ratings for green increase. Ratings for blue and red vary little with background luminance.

in saturation also occurs for bluish colors, this should cause a similar increase for the opponent category. No such increase was found. Instead the increase was observed exclusively for the characteristic object color. Therefore, the higher saturation alone caused by the distribution of hues in the stimulus cannot explain the results. Another sensory bias could occur because the distribution of the hues in the natural stimuli is not isoluminant. If the rating varies with the luminance of the stimuli, then the higher rating of yellow for the banana could occur because of brighter hues in the banana stimulus. However, the average yellow rating for the banana on the brightest background (27%) is still higher than the average rating of the disc on the darkest background (21%). Further, the banana image also contains values darker than the medium gray, which in the logic of the above argument would bias the rating towards lower levels.

If low-level sensory stimulus components alone are unlikely to explain the higher rating, maybe the cognitive knowledge of the banana as being yellow could explain the observers' response. In this case, an outline shape of the object should also result in a selective increase of the characteristic object color. However, in a control experiment with an outline of the banana we did not find such an effect.

Related to this issue is the question of whether the natural images of a banana have a sensory quality such that they really appear more yellow to the observer, or whether instead the banana images are merely judged more yellow based on a combination of sensory and cognitive factors. This study does not allow distinguishing between both possible interpretations. In another series of experiments with the same type of stimuli, observers made an achromatic setting of the fruits. In this paradigm, observers consistently made a setting where the achromatic mean is shifted in a direction opposite to the natural fruit color (Gegenfurtner & Walter, 2004). These results corroborate the idea that natural stimuli with a characteristic object color lead to a biased color appearance.

We suggest that the higher rating for the typical fruit color as observed in the present study may be due to cooperative interactions between top-down high-level cognitive memory color effects and low-level bottom-up chromatic distributions. The proposed cooperative interaction then selectively enhances the appearance of the fruit in its usual color. This effect should be taken into account when buying fruit and vegetables: bananas always appear more yellow than they really are, and the fresh green of the lettuce might be partly due to memory color.

Overall, we found that color scaling for natural images with a characteristic color is higher for this characteristic color compared to synthetic stimuli of a homogeneous color. These differences are most likely accompanied by a different color appearance of these objects.

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